

On God, Complementarity, and Decisions

*Consequences of a New Approach
towards Quantum Foundation*

by

Inge S. Helland

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Preface

The history behind this book started in 1971. I was a master student in physics at the University of Bergen. As it turned out then, the physics professor there said to me that he had no more to teach me, and he decided that I should visit the Niels Bohr Institute in Copenhagen for some months. But the stay there should only last for a little more than a month. Then I had a psychological breakdown and had to go back to my hometown Bergen. I will not go into details here, but a strong contributing factor behind this breakdown was professional. The physics that I was set to work with – formal quantum theory and S-matrix theory – was to me at that time too abstract and separated from what I saw as the real world. Could one make sense of a theory whose basis was that physical states were described by unit vectors in an abstract Hilbert space? At this point, I decided to quit physics.

After that, I had a year's study break. During that year, I had several different plans about how to continue my career, but finally I decided to go back to some sort of applied mathematics again. I graduated in statistics in 1973. Also, at that time I met my wife, Anne. We married in December 1973. We have two daughters, Annette and Siri, and 5 grandchildren.

In 1978 I got a position as a lecturer in statistics at The Agricultural University of Norway, and in 1983 I was appointed professor there. In 1986-87 I had a sabbatical in Edinburgh where I originally planned to learn more experimental design.

But, relying on my own ideas, I in fact spent more time there in a completely different area. A group in Norway headed by Harald Martens and Tormod Næs had done important work on a chemometric method called partial least squares regression, and I simply did not understand anything about what these people had been doing. I brought a big pile of their articles with me to Edinburgh and tried to digest these articles there. This took me some time, but finally I found a solution that I was satisfied with.

Since 1988 I have published several articles on partial least squares (PLS) regression, the best one being “On partial least squares regression and statistical models” in *Scandinavian Journal of Statistics*. PLS is an algorithmic method, but I found a way to associate it with a model of the kind that most statisticians now accept. The model has later been connected to the more general envelope models developed by Dennis Cook in Madison and his students.

In 1996 I finally realized that I basically was a theoretical person, and not a really applied statistician, and took a position as a professor in statistics at the University of Oslo. I stayed there until I retired in 2012.

I have enjoyed retirement. All my life I had felt that my break with fundamental physics in 1971 represented something that was unfulfilled, and I wanted to take this up again. My final goal has been, and still is, to build a bridge between the foundation of quantum theory and statistical theory. As I will explain in this book and have explained in more details in the technical article [5], I think that I now am on the way to reach this goal. But there are still unresolved problems.

There have been many attempts to find new foundations of quantum mechanics. Why do I think that my own approach is the best one? I have 3 answers: 1) The postulates, as formulated in Chapter 2 of this book, are relatively simple; 2) There are links towards statistics, both directly and via the proposed interpretation; 3) The postulates also give a foundation of Quantum Decision Theory; see Chapter 11.

One argument is via the resulting proposal for an interpretation of the theory. As is well known, there are many different interpretations of quantum theory. Theoretical physicists do not agree. Wikipedia lists 16 different, partially mutually excluding, quantum interpretations. My first proposal of an interpretation is a general epistemic one, an interpretation that has relationship with several other interpretations. Furthermore, this interpretation emerges in a natural way from my suggested foundation.

Concretely: A fundamental question is whether we shall look upon quantum mechanics as an ontological or epistemological theory. Is it a theory about the real world or about our knowledge of the world? After an extensive work on quantum foundations, I have ended up, at least basically, with a general epistemic or epistemological view: Quantum mechanics is about how an observer or a group of communicating observers achieve knowledge in a concrete context.

However, my basic theory is a purely mathematical theory, and it can in principle be interpreted in different directions. Look at the joint knowledge achievement of the group of communicating observers as mentioned above. In some physical contexts, this group may as well be the group of all relevant persons in the

world. Then their knowledge may be said to be objective knowledge, and in such situations, we may also say that we have an ontological interpretation of quantum mechanics. The epistemic and the ontological interpretations are merged together in this case.

This all should be compared to statistics, whose purpose is to give tools for obtaining knowledge of the world.

What do I mean by knowledge? In my view, it is knowledge about what I call theoretical variables, a fundamental notion. What should the observers be able to communicate about? As I see it, they at least should communicate about everything connected to the relevant theoretical variables. In this book, I will come back to this notion of theoretical variables several times.

Quantum mechanics is about knowledge in a specific context. This may be the case also in statistics, where the theoretical variables usually are parameters of a statistical model. But often, a statistical investigation has as its goal to arrive at conclusions that are valid also beyond the specific context.

Of course, there are differences between statistical theory and quantum theory; some of these seem to be connected to unresolved issues. A basic difference, as I see it, is that quantum mechanics in all cases must be seen from the point of view of an observer or from the point of view of a communicating group of observers. In statistics, this group always in principle may consist of all humans.

In statistics there has been large discussions related to what is called Bayesian, frequentist, and fiducial inference. In quantum theory, the distinction between what statisticians call aleatoric and

epistemic probabilities seems to be missing. As I see it, quantum probabilities can nearly always be seen as epistemic.

My hope is that my articles, and parts of the present book, will contribute positively to elucidating these questions. However, the focus of the present book will not be on the statistical problems, but on the wide consequences of my work on quantum foundations.

Some would say that my strong desire to build a bridge between two scientific societies, borders upon some kinds of religious endeavors. However, as it turned out, there are several purely rational arguments supporting the claim that such a bridge between the two scientific disciplines exist, something that I will come back to in Chapter 9 of the book.

Nevertheless, I may agree on the fact that in the process of my search I have had a basic religious faith, a faith that has helped me a lot.

In my scientific articles, I have been very careful not to mix religion and science, but I must confess that I always during my writings have had an idea of God in the back of my mind.

I firmly believe in the existence of God, and I reckon myself as a liberal Christian. This conviction has come gradually during the years that I have been working with fundamental science.

So, this, in addition to being a science book, will be a book based on what I call 'The God Hypothesis', the firm faith to the effect that there exists a God which has all knowledge that we can think

about. At the same time, He is perfectly rational in all His choices. My arguments for these conclusions, based upon my fundamental theory, will be given in Chapter 6 of the book.

Related considerations are given in the book 'Return of the God Hypothesis' [7] by Stephen C. Meyer, an interesting book. From step to step, relying upon a large amount of scientifically achieved results, Meyer has argued that there seems to be a higher intelligence behind everything. For instance, he considers the fine tunings of physical constants from the point of view that they give conditions under which life can exist, and in this connection cites the former physicist John Polkinghorne: 'Well, I don't say that the atheist is stupid, I only say that theism provides a more satisfying explanation'.

The challenges behind Meyer's endeavor to apply scientific methodology to uncover the viability of the God hypothesis are extremely large, but he claims that these challenges can be overcome. I see Stephen Meyer's discussion here as informative, but sometimes perhaps too detailed.

The great question is of course what we should mean by 'scientific methodology'. Scientists are human beings, and it by necessity follows that we also have our limitations. One of our limitations may be that we may have a too narrow 'methodology' concept, and any such narrow notion will be inadequate in this connection. For instance, pure empiricism will be useless. In his book, Meyer briefly mentions the possibility of designing experiments for theological assertions. I agree with him that any attempt in that direction will be futile.

Meyer ends his book by referring to Viktor Frankl's classic book 'Man's search for meaning'. This reference may also reflect some of my motivation behind writing this book.

Modern science has offered new possibilities for extending the 'methodology' notion. Quantum mechanics is a 100-year-old theory, and theoretical physicists have in all these years quarreled about how it should be understood. Nearly everybody agree that the formalism gives a rather complete theory that explains very many phenomena in the microworld. (Albert Einstein was an exception; he saw quantum theory as incomplete.) But, as mentioned, there exist several, partially mutually excluding interpretations of the theory.

This is a very confusing situation, but it gives us a possibility to propose a new scientific 'method'. What if we could find a set of simple postulates that implies the ordinary formalism, a set of postulates that 'everybody' could accept in some way? Then, exploring the consequences of these postulates to a very general setting, may be a new way of seeking the 'truth'. This will be attempted later in this book.

In [1] I have proposed some postulates that imply the quantum formalism, and I have started investigating the consequences of these postulates. The postulates have now been improved in [2]. Some of the postulates are very simple. I start with the notion of 'theoretical variables', a notion that can be made concrete in many different directions. I assume that these variables are connected to an observer or to a group of observers that can communicate over everything connected to these variables.

Common physical variables like position, spin component, momentum and energy may be such theoretical variables.

As already said, a standard view of quantum mechanics, now shared by many physicists, is that it is a theory about our knowledge of reality, not of reality itself. But then this knowledge must belong to the mind of some person, and/or to the joint minds of a group of communicating people. I interpret my theoretical variables also as relevant in this connection.

Mathematically, I only assume that, if s is a theoretical variable, and t is a function of s , then t is a theoretical variable. I also introduce the notions of accessible and inaccessible theoretical variable. As an interpretation, a variable is accessible if it can be measured with every desirable accuracy. But from a mathematical point of view, I only assume that if s is accessible, and t is a function of s , then t is accessible.

One crucial postulate should be mentioned: In every situation I assume that there exists a basic inaccessible variable ϕ or w such that every accessible variable is a function of w . In simple physical examples and in statistical applications such a w can easily be constructed. But as a ubiquitous postulate, covering all thinkable situations, it can most easily be understood by assuming the existence of a knowledgeable God, a God that at every moment of time knows everything that is known to every human being at that time. For a closer discussion of this, see [3] and Chapter 6 below.

Briefly, I will also mention one of the other postulates behind my theory. Quantum mechanics is based upon probabilities, and these probabilities may be found by what is called Born's rule. Several

scientists have proposed different postulates that should imply Born's rule. I have two postulates, one is from statistical science, the other runs as follows.

Assume that a scientist A is about to perform an experiment. During this process, he or she as a scientist has certain scientific ideals. These ideals can be modeled by an abstract or concrete individual D, who A sees as being perfectly rational. The notion of 'perfectly rational' is defined by the Dutch Book principle: No choice of payoffs in a series of bets shall lead to a sure loss for the bettor.

A rather thorough derivation of Born's rule from these postulates is given in [4]; see Chapter 3 below.

And a very reasonable interpretation of the last postulate is that there exists a God that can be perceived as being perfectly rational. Under some circumstances, and in connection to some questions, the individual D may be interpreted as God. The Born rule then gives probabilities connected to the answers to these questions. These probabilities are quantum probabilities, epistemic probabilities, and, in my opinion, depending upon the situation, may be interpreted as objective or subjective probabilities. They may be seen as probabilities from D's point of view.

The so-called QBists insist that quantum probabilities always should be subjective probabilities, but that is another discussion; see again Chapter 3. I agree in some sense with the QBists that the scientists that can calculate quantum probabilities must be said to have a special status, but these scientists should not necessarily be the same as the individuals who interpret the probabilities.

In my terminology, in the given situations discussed above, quantum probabilities must be interpreted as connected to the individual D. However, there is a very fine line to draw here. Most people have ideals that in no way are connected to physicists. These ideals inspire people in their decisions, and it is an open question how many of these decisions can be classified under the umbrella of Quantum Decision Theory, decisions for which quantum probability can be calculated and where such calculations are useful. See a discussion in Chapter 11.

The first three chapters of the book focus on my proposed foundation of quantum mechanics, particularly the aspects that I think may be related to statistics. This means that the important aspect of time development, as given by the Schrödinger equation, is not much discussed in the first chapters of the book. Chapter 4 discusses this, but the Chapter also depends on other, related approaches. A recent one by Jacob Barandes, which relies very much upon time dependence, is discussed in some detail. Steven Weinberg has given arguments to the effect that quantum states in general should be represented by density operators/ matrices, not by vectors in a Hilbert space, and has discussed transformations of density operators. But all these issues are technical, and the reader is in no way required to understand such technical issues. They are included here to satisfy the possible, more specialized reader.

In chapter 5, the epistemological interpretation is introduced and discussed from several angles. Chapter 6 gives my main argument for the existence of a divine being. Mathematically, there is a link here to two basic postulates in Chapter 2, postulates that in my opinion seem to characterize God. As I see it, such properties of God are independent of any choice of religion, and they should in

no way assume that the reader belongs to some Christian group or other religious group. Many people have an image of God; I have mine, but I claim that my image is consistent with the theory that I try to advocate here.

A mathematical argument is also the starting point of Chapter 7, but the final point there is psychological: According to my theory there exists a limitation on the set of variables that we can keep in our mind at the same time. Physically, this explains the outcomes of the well-known Bell experiments.

Chapter 8 reviews the complementarity concept, also for macroscopic applications. Several examples are given, also non-physical ones. Chapter 9 treats the connection to statistics and machine learning, and Chapter 10 briefly discusses the fact that what I say about a person's mind, also applies to the minds of scientists. An important notion here is a mental model, either quantitative or qualitative. Then Chapter 11 introduce the quantum theory of decisions as a special case of my theory. The final Chapter 12 gives my conclusions.

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Chapter 1

An Introduction

The modern technological development would have been impossible without science. In a certain sense physics lies behind all sciences, and it is impossible to discuss modern physics without touching quantum mechanics in some way or other.

The great American physicist Richard Feynman said once: 'If somebody claims that he understands quantum mechanics, he lies.' However, during the recent years essential new elements of understanding have appeared here.

Quantum mechanics is a fundamental physical theory that was developed in the beginning of the 20. century to explain a range of phenomena in the micro-world. This development is connected to important names like Bohr, Planck, Heisenberg, Schrödinger, Dirac, and von Neumann.

One crucial point is that quantum mechanics is a formalism, a set of calculating rules for how one can predict the outcome of certain experiments. These calculating rules have had a large success; they have been used for everything from small elementary particles to complex chemical and biological systems, and in every case where this has been tested, the predictions have been 100% in agreement with the results of experiments.

However, the great question is how one shall interpret the calculating rules. Here the physicists disagree, also today. During the recent years there has been held a long range of international

conferences on the foundation of quantum mechanics. A great number of interpretations have been proposed; in a term discussed below, one can say that many complementary views on quantum interpretation have been developed.

On two of these conferences recently there was taken an opinion poll among the participants. It turned out to be an astonishing disagreement on many fundamental and simple questions. One of these questions was: Is the quantum mechanics a description of our objective world, or is it only a description of our knowledge about reality? The first of these interpretations is called *ontological*, the second *epistemic* or *epistemological*. Up to now most physicists have supported the ontological interpretation of quantum mechanics, but versions of the epistemic interpretation have received a fresh impetus during the recent years.

My book 'Epistemic Processes' [1] is a research monography, but also a contribution to this debate. It relies on a general epistemic interpretation of quantum mechanics. An epistemic process can denote any process to achieve knowledge. It can be a statistical investigation or a physical measurement, but it can also be a simpler process. As I see it, such a process can often be related to quantum mechanics, even in some macroscopic cases. A pure quantum *state* can nearly always be associated with a focused question and a sharp answer to this question.

It is interesting that Weinberg [12] recently has proposed to base quantum mechanics upon *mixed states*, replacing state vectors by density matrices. The mixed states can still be based upon focused questions but giving answers in terms of probability distributions.

A specific interpretation consistent with the epistemic point of view is QBism, or quantum Bayesianism, see Caves et al. [10], but my interpretation is more general. The essential thing is that the observer plays a role which cannot be eliminated. This is crucial in my book [1], which also argues for a foundation of quantum mechanics from this basis. It is also crucial in all my articles.

Such an understanding can in my opinion be made valid for very many aspects of reality. We humans experience reality differently. Partly, this can be explained by the fact that we give different meaning to the concepts we use. Or we can have different contexts for our choices. An important aspect is that we focus differently.

Quantum mechanics, and experimental science in general, as I see it, can be expressed through what I call *theoretical variables*, variables defined or used by a person or by a group of communicating persons. The variables can be simple physical variables like energy, position, velocity, or spin component. Many such variables are what I call accessible, that is, it is possible to find accurate values for them by doing experiments.

By using group theory and group representation theory, I argue that one can study such a situation mathematically, and it seems to appear that an essential part of the quantum formulation can be re-derived under very weak conditions. The theory has now been further developed in several papers [2-6] and is reproduced in this book.

What we can conclude from such results, can also be related to how we should view the world around us. Empirically, the quantum formalism has turned out in physical situations in microcosm to

give a nearly all-embracing description. In addition, there has also in recent years appeared aspects of economy and psychology that can be explained by the quantum formulation. (See Khrennikov, [8] and Chapter 11 below.) Thus, the theory explained in this book, really seems to have important implications for reality in a very wide sense.

As an extreme consequence, it can be argued that for certain phenomena there exists no other possible views of reality than the subjective attached to an observer. This statement must be made precise to be understood in the correct way. First, it is connected to an ideal observer. Secondly, groups of observers that communicate between each other, can go in and act as one observer when a concrete measurement is focused on. When all potential observers agree on the answer to a measurement, this measurement must represent an objective property of reality. The objective world certainly exists; it is the *state* attached to certain aspects of the world that in some cases must be connected to an observer (or to several communicating observers).

The Born rule, that gives the probabilities in quantum mechanics, is in [1] derived from three assumptions: 1) a focused version of the statistician's likelihood principle; 2) an assumption about rationality, expressed by a Dutch book argument; 3) that the initial state is connected to a maximal accessible variable. The last assumption turns out to be unnecessary in a general version of Born's rule.

In 2), I do not necessarily assume that the experimentalist himself is perfectly rational. We can all do mistakes. But one can assume that the experimentalist, when making his decisions, has certain

ideals, and that these ideals can be modeled by something perfectly rational, that he looks up to.

Epistemic processes, at least the simplest of them, involve decisions in two stages: First a decision to choose a focus. Then a collection of data, and finally, there is an informed decision about what these data say about the phenomenon that we have focused upon. Traditional decision theory is only concerned with the last one of these decisions.

During the last decade there has been proposed and developed a new formal decision theory. This decision theory is inspired by quantum mechanics (Yukalov and Sornette, [11]), and is called Quantum Decision Theory.

Decisions can be based on knowledge, on beliefs, or both. Subjective belief cannot be enough.

In science, decisions are made within scientific groups, often international. However, there is also a need to think across groups, even across scientific disciplines. But it can be difficult to go into depth and at the same time keep a breadth that go across disciplines.

Decisions can be made by single persons or by groups of people. A group of people can agree to go collectively into a decision process, and they can make collective decisions on which actions should be done after the process is finished. Such decisions may take time.

But often we do not have much time for our decisions. Kahneman [7] has given an extensive account of the relationship between decisions based upon rapid thinking and decisions based upon slow thinking. Through many examples he argues that our minds are decomposed into two systems: System 1, where we take the rapid decisions necessary to function as human beings, and System 2, which is coupled in only through extra efforts, but which gives us, using a slower process, the possibility in certain cases to make more correct decisions.

In discussing these and similar questions, it can be useful to look at a concept from modern physics, namely the quantum mechanical concept *complementarity*. For a thorough discussion of complementarity in physics, see Plotnitsky [9]. The concept was originally introduced by Niels Bohr to describe what it is possible to measure physically, for instance that it is impossible at the same time to measure both the position and the velocity of a particle with arbitrary accuracy. But in various talks Bohr also looked upon extensions of the complementarity concept. Such extensions are indeed of great current interest.

Note that this quantum-related complementarity concept is slightly different from the concept of complementarity in the way it is used in everyday language.

In the present book I intend to discuss many consequences of all this. But it all depends upon my approach towards quantum foundation, an approach that I claim is simpler to understand than the usual formal foundation based upon vectors and operators in a complex Hilbert space.

The crucial assumption is that in every relevant given context and connected to an observer or a group of communicating observers exist two different accessible theoretical variables that in a certain sense are maximal as accessible variables.

This can be made precise mathematically and can be seen as the foundational principle for quantum mechanics that Anton Zeilinger [13] requested in 1999. In 2025, Anton Zeilinger gave a talk arguing for an epistemological interpretation of quantum mechanics at the famous Heisenberg conference, a conference where many different views were promoted.

Special relativity theory has as its principle that the theory is the same for all observers in uniform motion with respect to each other. In general relativity accelerated motion is also permitted. The theory of this book is the same for absolutely all observers or groups of communicating observers. Certain psychological properties of observers are included in the theory.

So, in this book, I will argue for a quantum foundation based upon a few simple postulates. The theory has many consequences, for instance arguments for the existence of a completely rational God, and an extensive decision theory. It is a hope that the arguments below will be convincing both to physicists, to other scientists and to interested laymen. I am open to anyone who will discuss these arguments.

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Chapter 2

Postulates and Immediate Consequences

Parts of this chapter are technical, but the chapter must be considered as very basic. All readers who are interested in the logical foundation of my theory, should have a look at the chapter. To help readers with little or no mathematical experience, the first part will be nearly without any technical details at all. This implies that some statements will be repeated.

A completely new approach towards quantum foundations is proposed in the books [1, 2] and in the articles Helland [3-13]. Those who want to look more on technical details, may consider my most recent articles [10-13], starting with, the last article [12]. The essence of all these articles will be formulated below. The most important mathematical proofs are given in [9].

The basis can be taken to be an observer C who is in some physical situation. He wants to investigate something, and to this end, he asks questions to Nature. These questions may be formulated in terms of what I call *theoretical variables*, a primitive and very general notion. In any situation I will say that there are theoretical variables, and some of these variables, say, s , t , ... may be related to the observer C .

The questions may then be of the form: 'What is s ?' or 'What will s be if I measure it?' In this book, I will concentrate on 'What'-questions. Questions starting with 'How' or 'Why' are largely outside the scope of the book.

Some of the theoretical variables are *accessible* to C , which means roughly that it is, in some future, in principle possible to obtain as accurate values as he wishes on the relevant variable. Other variables are inaccessible. Two physics examples of the latter are first the vector (position, momentum) of a particle at some time, and secondly the full spin vector of a spin particle, an imagined vector whose discretized projection in the direction a is the spin component in that direction.

From a mathematical point of view, I only require that if s is a theoretical variable, and t is a function of s , then t is a theoretical variable, and if s is accessible, then t is accessible.

So, these notions are very general, and can be made precise in many different directions. In a physical context, we just consider physical variables like position, momentum, energy, time, or spin component. Later, I will also look at statistical applications, where the theoretical variables are statistical parameters, which are called accessible if they can be estimated accurately using some estimation principle. And I will also look at general decision situations, where the basic theoretical variables are decision variables, that is, variables taking the value j if action number j is chosen or planned to be performed.

Theoretical variables may take a finite number of values, they may be scalars or vectors, or in general, they may take values in some topological space. The requirement that the class of theoretical variables/ accessible variables are closed under the process of taking functions, may seem artificial in certain situations, but it is convenient for the mathematical development.

As a special case, the class of theoretical variables are assumed to be closed under the process of taking one-to-one functions. This implies an equivalence relation among all the theoretical variables, and also among the accessible ones. In general, not all members of each equivalence class are of interest. In decision theory applications, only one member in each equivalence class is focused upon. In physics, the physical laws are formulated in terms of a small set of variables that are linear functions of each other. This is just related to the choice of scale, for example for a time variable, the reading of a clock may be said to be invariant to the choice of scale.

Taking one or a few members of each equivalence class, is certainly of relevance to classical physics and to the special relativity theory. In relation to general relativity theory, it might perhaps be useful in certain cases to consider a larger set of variables that are one-to-one functions of each other.

In my formulation of quantum theory, it turns out to be convenient to begin with to consider a large class of functions, say, Borel-measurable functions in the general case where the theoretical variables take values in some topological space. But mostly, we will think of the theoretical variables as scalars or vectors.

For discrete accessible variables, the answers to the 'What'-questions may be of the type ' $s=c$ ', answers that we in the physical case will associate with pure states of the physical system. More general answers may be of the form of a probability distribution over s , leading to mixed states.

Note that the event ' $s=c$ ' is equivalent to any event ' $f(s)=f(c)$ ' if f is a one-to-one function. And probability distributions over s are then equivalent to probability distributions over $f(s)$.

A crucial notion is that of a theoretical variable which is *maximal* as an accessible variable, that is, it is just possible to obtain accurate values by an experiment or an observation. A more extensive variable will be impossible to measure accurately. A more precise definition will be given below.

These notions are enough to formulate my main theorem: Assume in general that some symmetry conditions hold and assume a postulate stating that all accessible variables can be seen as functions of a large inaccessible variable w . Assume also, and that is crucial, that we have a situation where there exist two different maximal accessible variables. Then this is enough to ensure that an important first condition of the quantum formulation holds: There is a so-called Hilbert space associated with the situation, and to every accessible variable there corresponds a suitable operator in this Hilbert space.

This conclusion may seem technical, but the essence is this: Given a situation with some symmetry and with two different variables that are maximal as accessible variables – what Niels Bohr called two complementary variables, we can begin to talk about the quantum formalism.

In the case when the two variables take a finite number of values, the situation is even simpler: We do not longer need the symmetry assumptions; they are automatically satisfied.

For this finite case, several consequences can be derived from the mathematical theory: The set of possible values of a variable s is equal to the set of so-called eigenvalues of the corresponding operator. And s is maximal if and only if all these eigenvalues are distinct.

To an eigenvalue, there corresponds an eigenvector, and these eigenvectors can be seen, using the language of formal quantum theory again, as *states* of the system that we observe. Quite concretely, such a state can be seen as having an interpretation as a focused question together with an answer to this question: 'What is the value of s ?' / 'What will be its value if we measure it?' together with an answer of the type ' $s=c$ '. This simple interpretation assumes that s is maximal. In general, the so-called eigenspaces have such an interpretation.

All this theory leads us a long way on the road to 'understand quantum mechanics', a problem that has occupied physicists since the formalism was invented 100 years ago.

The above characterization of accessible and inaccessible variables is related to implications of the theory. The theory itself is purely mathematical, can be applied in different directions, and the terms 'accessible' and 'inaccessible' are also just primitive notions of the theory. To repeat, two other ways that the theory can be applied, are 1) Quantum Decision Theory, where the variables are decision variables; 2) statistical inference theory, where the variables either are statistical parameters, future data, or a combination of parameters and data.

From a mathematical point of view, it is only assumed that if s is a theoretical variable and $t \equiv f(s)$ for some function f , then t is a theoretical variable. And if s is accessible, then t is accessible.

Variables may be called 'equal', see below, if they are in one-to-one correspondence. Then they contain the same information. Variables that are not in one-to-one correspondence, may be called 'different'.

The main application in this Chapter is that these variables are theoretical variables coupled to a physical situation. If necessary, an observed variable can be modeled statistically as a theoretical variable plus some random error. Following Zwirn [15, 16], every description of reality must be seen from the point of view of some observer. Hence, we can assume that the variables also exist relative to C . I differ from Zwirn by the fact that I also allow several communicating observers.

It is important that observers may communicate. The mathematical model developed in the articles mentioned above is equally valid relative to a group of people that can communicate about the physics and about the relevant theoretical variables.

This gives a new version of the theory, a version where all theoretical variables are defined jointly for such a group. The only difference here is that, for the variables to function during the communication, they must always be possible to define them in words. To be precise, I should also state what the observers should be able to communicate about. In my theory, this is everything that is related to the relevant theoretical variables.

In the two examples mentioned above, the position/momentum case and the spin component case, there are also maximal accessible variables: In the first example this is either position or momentum, in the second example they are the spin components q_a in some direction a .

From a mathematical point of view, an accessible variable s is called maximal if there is no other accessible variable t such that $s = f(t)$ for some non-invertible function f . In other words, the term 'maximal' will then be seen to be maximal with respect to the partial ordering of variables given by $u \leq s$ if and only if $u = f(s)$ for some function f . Note that according to this partial ordering, variables are 'equal' if they are in one-to-one correspondence. Then they contain the same information.

Variables that are maximal, different, not in one-to-one correspondence, but having similar ranges, represent my interpretation of what Niels Bohr called complementary variables; see later for a closer discussion of this aspect.

A fundamental assumption in my theory is that there exists a basic inaccessible variable w such that all the accessible variables can be seen as functions of w . In the two mentioned examples, w can be taken as the vector (position, momentum), respectively the imagined full spin vector. A crucial symmetry assumption is that there is a group K acting on w ; these technicalities will be specified later.

I should remark here that my basic article [9] comes in two versions. In the arXiv version, some simple category theory is used. This might be the beginning of something important. But the version in