

Disagreement and Language

A Formal Approach to Choosing Our Words

By

John Craven

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Preface

This book is the product of diverse circumstances. As an economist, I have had an almost life-long interest in social choice theory, which examines ways of trying to resolve disagreements about the benefits of economic policies. Alongside, I have performed as an amateur musician, singing music from West End shows to Tudor madrigals. Then grandchildren arrived and I watched them learning their languages.

Perhaps the greatest serendipity comes from my decision to study for a master's degree and then a doctorate in philosophy at King's College, London because I could not face returning to economics after trying to run a university for sixteen years. The subject had changed too much, and perhaps I had as well. I finally homed in on my thesis topic in the philosophy of language and abandoned the idea of finding something original to write about vagueness. The youngest generation showed me that we learn to use language and often communicate well despite some obvious disagreements amongst those from whom we learn. I wondered about the theory behind our attempts to reconcile contested evidence, using music as my main example. So JAZZ met social choice theory.

Studying philosophy in my mid-sixties means that I do not come to this project versed in philosophers' language. That is not a criticism of philosophers, economists have their jargon too. But it means that some might want to rephrase what I say in the terminology of their subject. If several people do so, they might disagree, indirectly proving my point.

I am very grateful to Professor Eliot Michaelson who very patiently and helpfully supervised my dissertation. I hope that he learned something from my struggles to make sense of my ideas and that I have repaid him at least to the extent that he has a successful student who has contributed to several learned journals. My other recent influence has been Professor George Burrows at the University of Portsmouth, who is an inspiring choir director. His extensive knowledge of music of many genres led to several very helpful suggestions for my thesis and for this book. I enjoyed playing a minor role in his AHRC project *Musical Theatre and all that Jazz* and it was gratifying that participants agreed that they do not always agree.

Of course, many others have influenced me. I am grateful to them all and hold them responsible for none of my errors and misunderstandings. But primarily my inspiration and support now comes from Laura, my wife of fifty years, Matt and Becca, our children, and in the last eight years from their children, Rosa, Calder, Vito, Franco and Arden, to whom this book is dedicated. Rosa gets the name-check only because she was the first on the scene.

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January 2026

About the Author

John Craven has been successively an undergraduate at King's College in Cambridge, a Kennedy Memorial Scholar at MIT, a lecturer, professor of economics, faculty dean and deputy vice-chancellor at the University of Kent and vice-chancellor at the University of Portsmouth. He founded the University Alliance mission group of universities. After retiring in 2013, he gained an MA and then PhD in philosophy at King's College, London where he is now a Visiting Research Fellow. He has been a member of the boards of a national museum, a theatre, educational institutions, two cathedrals, charities, a housing association and of the Archbishops' Council of the Church of England. He was awarded a CBE by Queen Elizabeth in 2013. He has written three other books, numerous journal articles and more strategy papers than is good for anyone.

Chapter 1

Prelude

Rosa is learning her native language. By the time that our discussion opens, Rosa is reasonably fluent in the basic rules and conventions of grammar and she has a vocabulary that allows her to participate in day-to-day activities. Typically, she will have learned these basics as a young child from those with whom she interacts at home or in other environments. These influences remain important as she moves into another stage of her language development. This involves learning how to use kinds or collective concepts or genres or named categories. By using these, she can communicate about many individual items using a single term.

Our central example – but it is only an example – involves genres of music, such as JAZZ, BLUES, CLASSICAL, OPERA and so on. Once she has some familiarity with the use of these genre concepts, Rosa might say “JAZZ and BLUES have their origins in the south of the USA around 1900” or “the development of JAZZ was an important influence on race relations between the two world wars” or “OPERAS have great scores but terrible plots” and then discuss these issues and opinions with others. She does not need to recite a list of the music that she regards as JAZZ or BLUES or OPERA during those discussions, and, crucially for the questions that we ask here, it might be that those with whom Rosa communicates do not share all of her opinions about everything that she includes in JAZZ. But they still have sufficient common ground that they can hold a reasoned discussion without anyone needing to ask, “Tell me exactly what you regard as JAZZ.”

Although musical genres form our main example, very few named musical compositions appear in these pages, and musicologists or other aficionados might criticise what is implied here about the ones that do. But that only adds to the relevance of our agenda because it is evidence of disagreement, unless all the critics can confirm that they always agree. That seems to be improbable in any creative discipline.

Rosa learns to use these concepts by absorbing the ways in which others use them. These others might include specific individuals whom we identify as Rosa's influencers, or she might develop an impression of how public opinion uses genre concepts without being able to attribute the opinions to specified people. She might read about the history of JAZZ and other genres and, of course, she might listen critically to music or perform herself. She has many possible sources of evidence from others and from her own reflections and it is realistic to suppose that these sources do not always agree. The challenges that are examined here involve Rosa's attempts to find a way of reaching a conclusion about genres which, in some as yet ill-defined way, represents an acceptable synthesis of the evidence.

Psychologists, education experts and other professionals have put forward many insights and hypotheses that relate to the ways in which people learn to use the language of their community. That is not our agenda. We start from a different place, beginning with the observation that communities and individuals need to find ways of coping with many forms of disagreement. Perhaps the most obvious instances outside the development of language are the elections that many countries, communities and organisations hold to fill positions of authority. An election is needed because there is rarely agreement about the identity of the best candidate.

An electoral system consists of the rules whereby the expressed views of voters are translated into a result. There are many such systems, including plurality voting (first-past-the post that is used in many UK elections) and various interpretations of transferable and alternative votes. Those who are elected join parliaments or committees that also need ways of reaching a conclusion in the face of disagreement about the relative merits of rival proposals. These bodies often vote, using their own rules for reaching a result.

Individuals face disagreement and seek ways of coping with it. This can happen in the day-to-day life of a family, with the passing observation that disagreement does not necessarily involve contentious dispute. Perhaps more grandly, anyone who wants to make a consequentialist ethical judgement – for example by following the routes set out by utilitarian philosophers – needs to take account of the fact that there is likely to be disagreement about the relative merits of two actions or states of the world. In many family discussions and in most ethical comparisons, the outcome favours some people over others: a practical resolution creates winners and losers.

Those who are trying to cope with disagreement might consider using a particular electoral system or other method, such following the majority opinion when they make an ethical judgement or taking turns to choose the film that the family will watch. They can then assess whether the resulting outcome is satisfactory whatever the voters say, whatever the interests of citizens might be or whatever family dynamics ensue. If it is unsatisfactory, they might ask whether any other method could be consistent with the principles that define ‘satisfactory’, or they might assess whether there are good reasons why the extent of disagreement is likely to be sufficiently limited that the possibility of reaching an unsatis-

factory outcome is rare or even non-existent. Or someone who is settling a family disagreement or making a judgement might argue that there are times when they should recognise that the strong personal interests of one person should outweigh the opinions of others. That, arguably, is what liberals do, at least sometimes.

Our central theme involves the recognition that Rosa's sources of evidence do not all agree about the genre locations of the music that she considers. She faces disagreement, and we can identify the three questions that are suggested in the previous paragraph:

question 1: Is there a risk that Rosa is not satisfied with the outcome if she uses a particular method for resolving disagreements that arise?

question 2: If the answer to question 1 is "yes", is there some alternative method which eliminates that risk?

question 3: Can Rosa be confident that the extent of disagreement within her evidence is sufficiently limited that the possibility of an unsatisfactory outcome is removed?

Our examination of these questions involves modelling. This inevitably gives a stylised interpretation of how Rosa goes about meeting her challenge. We start when Rosa is new to the topic and relies only on evidence from influencers who are more expert than herself in musical matters. Their evidence is likely to be constrained by the conventional uses of language that are often summed up in dictionary definitions of the genres. For example, there might be disagreement about whether Handel's *Semele* is an OPERA or an ORATORIO, but any dictionary definition or reference book would exclude the suggestion that *Semele* is JAZZ. Likewise, there has been disagreement about whether Gershwin's *Rhapsody*

in Blue is JAZZ or CLASSICAL, but it is certainly not an OPERA according to dictionary definitions. Rosa can learn these constraints on genre locations from those whose opinions she uses as evidence, even if she does not directly consult dictionaries herself. None of her influencers will argue that *Semele* is JAZZ or *Rhapsody in Blue* is an OPERA and so Rosa has no evidence that would support these unconventional opinions.

After Rosa has used evidence from her influencers to decide her own genre locations for an initial set of pieces of music, she can reflect on those outcomes to see whether they are satisfactory. By that time, Rosa has sufficient personal experience of music that she can begin to make her own judgements. She begins to form an opinion about whether two compositions are similar enough that they can be co-located in the same genre, or whether their contrasting features lead her to think that they should be located separately. She might also have read some authoritative works, so that she can include evidence about historical origins or social influences. In SHORT, Rosa develops a private opinion that she can include as evidence alongside that from her influencers. Also, by this time, Rosa might have gained a perception of public opinion as it is expressed widely within her community, including by people who have no significant claims to musical expertise and whom she cannot always identify. Instead, she knows roughly how frequently opinions are expressed.

An important aspect of the challenges that Rosa faces is that she is likely to use the evidence to locate music into genres sequentially. She considers one piece of music at a time and after she has done so for a while, she reviews her decisions and possibly revises them. As well as considering the ways in which she might revise her decisions at the end of this first phase, we examine the conse-

quences of assuming that Rosa does not have time or inclination to circle back during the first phase to revise earlier decisions each time that she considers another piece of music.

Rosa can use the lessons learned from her review of the first phase when she later expands the range of music that she is considering. Her lifelong learning can include a continuing interest in the topic as she discovers new music, but the difficult issues are likely to be identified fairly early. These are our main focus.

This outline of Rosa's language challenges raises several issues, not least about how Rosa might derive an outcome from the evidence. What, in a sense, are her equivalents of electoral systems? A related issue concerns Rosa's view about what constitutes 'satisfactory' when she is reviewing the first phase. What are her equivalents of the principles that might be used in making ethical judgements?

Once we have pursued these issues, we turn to variations on the theme. The first involves the possibility that Rosa gives some special status to evidence from composers or others who have a close relationship with a piece of music. In the light of other evidence, she might not always follow these opinions, and we ask whether this rejection can be consistent with principles that might define 'satisfactory' from Rosa's point of view. Perhaps surprisingly, the exploration of this question can be transferred to challenges in which Rosa is deciding how to allocate people to schools of thought or to demographic categories. For example, she might consider rejecting the idea that everyone who identifies themselves as CHRISTIAN in a national census is in that category given that the numbers who self-identify can far outweigh the numbers of regular churchgoers. More topically perhaps, does she accept self-identification by gender, given that public opinion is sometimes strongly

opposed? Or she might try to make sense of disagreement about philosophers' membership of schools of thought, including, but not always following, the opinions of the philosophers themselves.

The transfer of methods is possible because a composer can be thought of as the 'owner' of their music and a citizen can be thought of as the 'owner' of their religion or of their gender. The conclusions and interpretations might, of course, not be transferable even if the methods of analysis are. There could be contexts in which Rosa regards ownership as definitive, and contexts where she does not. She might follow every self-identity when allocating people to religious categories but reject some composers' opinions about the genre of their music.

A second variation on our theme arises when there is a feature that reduces the extent of disagreement sufficiently to allow a positive answer to question 3. We use as an example the possibility that everyone in a community agrees that curry dishes can be ranked from mildest to hottest, with no disagreement about their grading by spiciness. Even then, Rosa's evidence might include disagreement about which dishes are described as *MILD*, *MODERATE* or *HOT*. Does the agreement about the grading of the dishes imply that Rosa can face contested evidence but still find an acceptable outcome wherever her influencers place the boundaries between the categories?

The exploration of many of these aspects of Rosa's language challenges involves the use of techniques from **social choice theory**. This theory involves a systematic and abstract approach to the issues that can arise in reaching a satisfactory outcome given evidence that involves disagreement, including a discussion of the principles that lie behind the word 'satisfactory'. It has origins

in consequentialist ethics and is frequently used in discussions of electoral systems. As far as the author is aware, social choice has not been used in the context of language development, and some of our conclusions might be fed back to inform other applications of social choice. That, arguably, is a bonus beyond the examination of Rosa's language challenges.

Chapter 2

Modelling Rosa's Challenges

2.1 Evidence

Our approach to examining the ways in which Rosa copes with disagreement when she is trying to define her own uses of kinds or collective concepts or named categories involves building models. Many economic theorists do little else. Model-builders try to distil the essence of a problem, formalise it and then use that formal structure to extract conclusions. Debates about the usefulness of a model then assess ways in which its conclusions are relevant to actual circumstances that are likely to include complexities that the model assumes away.

Rosa has been named simply to avoid confusion or circumlocution because there can be many other people involved in language challenges. Rosa's evidence comes at least in part from identifiable others in her community whom we label as her **influencers**. If she is trying to locate music in genres, her influencers might include musicologists, experienced critics, composers or performers. The simplest assumption is that these influencers just supply evidence about the genre locations of the compositions that Rosa is considering. None of them has a personal interest in any of the compositions and we assume that they are independent of each other so that there is no connection between the evidence that is supplied by different influencers. In later versions of her challenges, Rosa can be influenced by people, such as composers, who have a personal interest in compositions. Then we need to allow that not all influ-

encers express an opinion about the location of all the compositions that Rosa is considering. After all, a composer might have died before others have devised their works.

These influencers are not necessarily her only sources of evidence. As Rosa experiences more music, she develops her knowledge and understanding of JAZZ and of other genres and she might develop her own private opinion about whether some piece of music is JAZZ. This does not directly depend on the opinions of her influencers and might be based on her own reflections about the similarity or otherwise of one piece of music to others that she has already – at least provisionally – located in a genre. Or she might be swayed by historical evidence that involves the connection between a performance or a performer and the music of slaves or former slaves that is widely agreed to be the origin of music now called JAZZ. Or she might have been involved in discussions which give her an indication of public opinion in general, without being able to identify specific influential fellow members of her community.

Rosa might consult dictionaries. These rarely define JAZZ by listing the music to be included – although on-line resources such as *Wikipedia* provide lists of, for example, JAZZ ‘standards’ that are widely regarded as JAZZ. Instead, dictionaries try to encapsulate conventional ways of identifying music as JAZZ, such as

“A type of popular music originating ... among African Americans in the southern United States, typically performed by ensembles and broadly characterized by regular forceful rhythms, syncopated phrasing, modifications to traditional instrumental tone and pitch (such as the use of blue notes), and improvisatory soloing”. (Oxford)

"A type of modern music originally developed by African-Americans, with a rhythm in which the strong notes often come before the beat. Jazz is usually improvised. (Cambridge)

"American music developed especially from ragtime and BLUES and characterized by propulsive syncopated rhythms, polyphonic ensemble playing, varying degrees of improvisation, and often deliberate distortions of pitch and timbre." (Merriam-Webster)

Rosa can test a piece of music against these definitions and use that as a part of her private opinion.

Our terminology is that Rosa has a **profile** of evidence from some or all of these sources and reaches an **outcome** based on that evidence. We could leave this as a black-box statement, without enquiring how she derives the outcome from the evidence, but our agenda is to build a model that purports to catch the essence of the ways in which Rosa links the evidence and the outcome, and to assess the implications. We ask, "does Rosa regard the outcome as acceptable given the evidence that she has?", and "how would the outcome differ if her evidence changes and are those differences acceptable?".

This is the main thrust of our agenda and we follow it in various directions.

2.2 Disagreement: an example from elections

There are other aspects of the life of a community that require a conclusion when there is disagreement. We can take advantage of

work that has been done to model these areas. An obvious example arises in elections where the winner is decided using an electoral system that is a set of rules that are applied to voters' statements in order to reach an outcome.

Much has been published on the theoretical analysis of elections and some of the approaches and techniques used there can be transferred to our agenda. There are obvious differences. Many elections select one candidate as the winner; a language challenge determines whether some composition is included in a named genre. Timescales differ too: language development can be a part of life-long learning whereas a community is likely to want an electoral system to *operate* quickly. But, even so, underlying both agendas – and others referred to later – is the question of how to reach a conclusion in the face of evidence in which there is disagreement.

Electoral systems differ from one another because they can give different outcomes from a given profile of evidence. A plurality (or first-past-the-post) system can give a different winner from one that uses transferrable votes or from one that assigns points in accordance with the voters' stated preferences and defines the winner as the candidate with the largest number of points. The temptation to vote tactically is well-known in a plurality system when there are more than two candidates and it is possible to show that similar possibilities can arise in systems that use transferrable votes or that are points-based. But there are many possible electoral systems, including both those in use somewhere and others that are feasible but not implemented anywhere. There are so many possibilities that it is difficult to assess them one-by-one. Instead, Allan Gibbard (1973) and Mark Satterthwaite (1975) published papers that answered the question "Is there any accept-

able electoral system that is immune to tactical voting, whatever preferences people hold?"

The criteria that define an 'acceptable' electoral system clearly include

- the electoral system should follow the consensus when the voters all give identical evidence; and
- there are circumstances in which every voter should be able to affect the outcome by alone changing their stated evidence. Everyone has a chance of being a 'marginal voter'.

The required immunity to tactical voting is that there are no circumstances – no profile of evidence from the voters – in which anyone has an incentive to state a preference other than that which they truly hold. The Gibbard-Satterthwaite question seeks to avoid any risk that tactical voting can be advantageous to any voter. In any actual election, most obviously in any with a very clear-cut winner, tactical voting could be ineffective without negating the risk that arises when opinion is more evenly divided. Given these requirements, the authors show that the answer is "no" when there are more than two candidates. Every electoral system is open to tactical voting in some circumstance.

The transferable lesson is that there are two ways of examining how Rosa responds to a challenge. We can examine the consequences of using some rule or formula or algorithm that governs Rosa's conclusion – such as that she follows the majority opinion of her expert influencers. Alternatively, in the spirit of the Gibbard-Satterthwaite question, Rosa can ask "Is there any acceptable way in which Rosa can meet a language challenge whatever evidence she has?"

As we shall see, the answer depends on the nature of the challenge and on the criteria that Rosa uses to define ‘acceptable’. Again, the question seeks to avoid the risk that there are profiles of evidence for which Rosa finds the outcome to be unacceptable.

Theoretical aspects of elections, including proofs of the Gibbard-Satterthwaite theorem can be examined using techniques from social choice theory. The name reflects how its early proponents tended to apply it to situations such as elections in which a community or a committee wants an outcome – a social choice – despite underlying disagreement. In Rosa’s challenges, the outcomes are hers, but we stick with the name of the theory given the frequency of its use in many areas of discourse and the extent of the theoretical contributions from which we might draw. Chapter 5 explores methods of social choice theory as they have been developed in a now voluminous literature.

2.3 Reconciliation, controversy and truth values

Not all evidence conflicts and, when we say that there is disagreement between two or more sources of evidence, we imply only that the evidence is different. We are not here concerned with conciliatory efforts that might be made to bring about agreement. In SHORT, Rosa’s challenge does not involve attempts to negotiate between her influencers. When they disagree, she wants to know whether there is an acceptable compromise. As we shall see, she might not find one.

Despite quoting examples that can be the subject of major controversy, ‘disagreement’ as we use it here is not intended to imply that people react to each other in a heated or even argumentative way. The term implies only that evidence can differ between

sources. The implications for Rosa's challenge are the only consequences of conflicting evidence that interests us here. We leave other consequences of disagreement to lawyers and to Jonathan Swift's big-enders and little-enders.

Disagreement between influencers might arise because experts have different views about, say, whether George Gershwin's *Rhapsody in Blue* is JAZZ or CLASSICAL music. If one influencer states "*Rhapsody* is JAZZ" and another states "*Rhapsody* is not JAZZ" it is not (or not necessarily) the case that one makes a truthful statement and one does not. If truth values are wanted – which is not our purpose – the nearest we can achieve is to say that "In the opinion of the first influencer, *Rhapsody* is JAZZ". The influencers can agree that this statement is true. They can also agree that "In the opinion of the second influencer, *Rhapsody* is not JAZZ" is true. Their agreement that both of these personalized statements is true does not help Rosa, who still faces a challenge to find a compromise given the evidence of their underlying disagreement.

2.4 Other challenges

Some of the issues that we consider here might have arisen at an earlier stage of Rosa's language development. Most children learn to identify colours long before the challenge of identifying JAZZ arises, and a young child might find that one parent regards a particular piece of fabric as blue and the other regards it as green. We leave open whether any part of what follows can be applied to this early learning. Our narrative generally starts later in Rosa's journey.

The classification of music into genres is one example. A related challenge involves the classification of performers into categories

such as JAZZ musicians, BLUES singers or CLASSICAL pianists. Libraries and bookstores classify and locate books or authors according to different classification schemes. Philosophers categorise theories and other philosophers and it is difficult to assert that they always agree. Art and artists can be classified into genres or schools. There was debate amongst the ‘candidates’ in nineteenth century France as to who counts as an impressionist – and the debate included the opinions of some of the artists themselves.

Evidence about natural kinds is likely to involve a chemical formula or genetic profile to which objects must conform if they are to be included, at least within the scope and accuracy of available methods of analysis and measurement. There might not be much disagreement in these examples, and so they are not of great interest here. But medical diagnoses can involve important disagreements that are revealed by seeking a second opinion about the disease category of a patient even though each opinion is informed by the same test results. The test results are evidence, but are not always definitive, leaving scope for the inclusion of evidence based on the opinions of experts. The fact that ultimately a definitive test will be devised – or that the most accurate test is available only in an autopsy – is not helpful in diagnosing the category of disease that affects a live patient today. Someone, possibly the patient, is likely to need to adopt one out of two or more possible diagnoses. That person wants to resolve disagreement.

2.5 Formal structure

Model-building requires some definitions and some notation. Our narratives involve the **location** of **objects** to **categories** or more formally to the extensions of the concepts that lie behind the cate-

gories. Categories are denoted in SMALL capitals and might be referred to as genres or kinds such as JAZZ, CLASSICAL MUSIC, IMPRESSIONISM, CUBISM, INFLUENZA, LYME DISEASE. **Rosa's set** consists of the objects that Rosa is considering for location in one of the categories albeit over a period of time.

In some cases, we refer to human **subjects**, rather than objects, as for example in the allocation of philosophers to schools or citizens to demographic categories. Unless they are named, objects or subjects are always denoted by lower case Roman letters x , y , z , etc. Rosa's influencers are denoted by lower case Greek letters α , β , γ , etc.

Partial and general challenges

We distinguish two forms of challenge:

- In a **partial challenge**, Rosa is concerned with a single category and wants to decide which objects or subjects to locate in it. We would generally suppose that the objects that she locates in, say, JAZZ or IMPRESSIONIST have some features in common, whereas the objects that she does not locate there might be very disparate. They are located in the complementary category, not-JAZZ or not-IMPRESSIONIST.
- In a **general challenge**, there are several named categories and Rosa wants to decide where to locate each object or subject. In a musical challenge, she wants to allocate compositions to genres such as JAZZ, BLUES, CLASSICAL, OPERA, etc. We could allow that she uses a residual category NONE-OF-THE-ABOVE with the proviso that, unlike the named genres, but like not-JAZZ in a partial challenge, there is no implication that objects in the residual category share any

characteristics. Generally, we do not allow explicitly for such a catch-all category.

The terms ‘partial’ and ‘general’ have echoes in economics where the examination of a single market involves a partial equilibrium using evidence about that market alone. For example, the price of tea depends only on the demand for and supply of tea. A general equilibrium involves evidence from many markets that can be inter-related in many complicated ways. The price of tea also depends on the price of coffee (a substitute, at least for some people), the wages of tea-pickers and tea-drinkers, the cost of kettles (a complementary good) and so on. In turn the price of coffee depends on the price of tea, so that in effect everything depends on everything else and prices are all determined simultaneously. The two forms of model are useful in examining different questions here, as they are in economic theory.

2.6 Aggregators

Rosa’s challenge is a classic **aggregation problem**. She has a profile of evidence from multiple sources and she associates an outcome with that profile. She would derive an outcome, possibly different, if the profile of evidence was different. In mathematical terminology, Rosa uses a **function** that maps profiles of evidence to outcome locations of objects or subjects. In an election, an electoral system plays the role of a function, and an electoral system is usually expected to define a winning candidate given any logically possible profile of evidence from the voters. If it didn’t, chaos might ensue. We can make the same assumption in Rosa’s challenge. She needs to be able to reach an outcome whatever evidence she faces. In the absence of a specific term for the function that

maps evidence to outcome in a language challenge, we refer to it as an **aggregator**. An aggregator in a language challenge plays a parallel role to an electoral system in a voting context or as a social welfare function in economics.

Some aggregators can be defined by the use of an algorithm that gives an outcome from whatever evidence Rosa has. For example, we refer frequently to a **majority aggregator** which is based on the algorithm that Rosa follows the majority opinion about the location of each object or subject when the evidence includes only two locations for it. A **preponderance aggregator** plays a similar role when the evidence is more varied by directing Rosa to follow the most frequently expressed opinion of the three or more that appear in the evidence. A very simple aggregator – though not a very attractive one in many circumstances – implies that Rosa's outcome is always the same as the evidence from a single dominant source. But in general

an aggregator consists of a list of possible profiles of evidence and the outcome that Rosa associates with each.

The aggregator says what Rosa does, or would do, according to the evidence that she has. Electoral systems tend to use algorithms (first-past-the-post, single transferable vote, Borda score...) but all that is formally required to guarantee an outcome is that the returning officer has a (long) list of possible patterns of voting, each associated with a winning candidate. Whether such an electoral system would be regarded as sufficiently transparent that it is acceptable is another matter.

These lists would be long because each list contains an entry for each possible profile of evidence. Furthermore, there are many